The Time-varying Risk-Return Tradeoff in the Long-Run

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Abstract

Lundblad(2007,JFE) shows that the risk-return tradeoff is unequivocally positive with a two-century history of equity market data. A further examination of the relation with the UK monthly stock returns from 1836 to 2010 produces rather weak risk-return relation. I show that the risk-return relation is mostly positive but varies considerably over time based on a new nonlinear ICAPM with multivariate GARCH-M terms with the time-varying risk-return tradeoffs and hedging coefficients. The often observed negative risk-return relation is also statistically insignificant with the 95% confidence bounds. The hedging coefficients also vary significantly across time. This complex nonlinearity seems to be the main culprit of the weak risk-return relation.

Keywords: Time-varying Risk-Return Tradeoff and Hedging Coefficient, ICAPM, State-Space models with GARCH

JEL Classification: G12, C15, C22

1 Introduction

Mainstream asset pricing theories such as the CAPM or ICAPM implies a positive risk-return tradeoff. While the risk-return tradeoff is fundamental to finance, the empirical evidence has been rather inconclusive. For example, French, Schwert, and Stambaugh (1987) and Scruggs (1998) find a positive relation between the expected excess market return and conditional variance, whereas Glosten, Jagannathan, and Runkle (1993) and Scruggs and Glabadanidis (2003) find either a negative or insignificant relation.

Using the nearly two-century history of the equity market data, Lundblad (2007) finds that the risk-return relationship is positive and significant regardless of GARCH specifications. His Monte Carlo analysis shows that researchers need more than 100 years of data to estimate the relation between the market risk premium and conditional volatility with any precision. He argues that the weak empirical relationship found from the previous research may be viewed as a statistical artefact of small samples and that in the GARCH-M context, one simply requires sufficiently a long span of data in order to detect this relationship.

In this paper, I re-examine the issue with continuously compounded UK stock and bond returns with a two-century of data from 1836:01 to 2010:12. I first estimate a univariate GARCH-M model as in Lundblad (2007). I first present evidence that a long span of time-series does not seem to guarantee a significantly positive risk-return relation in the univariate model. I find rather weak evidence for the positive relation. I also extend his specification and estimate a version of ICAPM with two factors employed in Scruggs and Glabadanidis (2003) and find that even this two-factor model produces rather weak risk-return relation.

Theoretically, the risk-return tradeoff can be time-varying with any sign. In most popular asset pricing models such as the CAPM or ICAPM, the source of time-varying

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relation could be time-varying positive risk aversion. However, the negative relation can be also justifiable. For example, Whitelaw (2000) shows that the negative relation exists when the market excess returns acts as a proxy for hedging components in a regime switching consumption based model. Furthermore several empirical studies such as Whitelaw (1994) and Brandt and Qiang (2004) show that the risk-return relation is unstable.

In this paper, I develop a new nonlinear two-factor ICAPM with bivariate GARCH-M models to incorporate this unstable risk-return relation. Because it is not easy to determine the sources of time-varying relation in the long run data due to historical data limitation, I propose an econometric model based on state space model with GARCH with the latent time-varying risk-return tradeoffs and the latent hedging coefficients.[1] In summary I find that the risk-return relation is largely positive across time. Even when the point estimate indicates the negative relation, it is not statistically different from zeros with 95% confidence bounds. The hedging coefficients also vary a lot across time with negative signs in most cases. I argue that the time-varying risk-return trade-off is the main reason for the weak risk-return relation.

The remainder of this paper is organized as follows: Section 2 provides the theoretical framework and the empirical models for the risk-return relationship. Section 3 presents the econometric methodology to estimate the proposed nonlinear ICAPM. In Section 4, the historical data and the sources are discussed and time-series evidence on the risk-return relationship with the usual conditional CAPM and ICAPM are provided. In Section 5, I present empirical results with a nonlinear ICAPM with the time-varying risk-return trade-off and the changing hedging coefficient. Finally, Section 6 concludes.

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[1] In the previous version of this paper, I also estimate the similar model with time-varying risk-return trade-off and constant hedging coefficients. It produces largely similar results. The results are available upon request.

2   Risk and Return in Equilibrium

Merton (1973) derives the dynamic risk-return trade-off between the conditional mean of the return on the wealth portfolio, $E_t[r_{M,t+1}]$, in relation to its conditional variance, $\sigma_{M,t}^2$ and the conditional covariance with variation in the investment opportunity set, $\sigma_{MF,t}$:

$$E_t[r_{M,t+1} - r_{f,t}] = \left[\frac{-J_{WW}W}{J_W}\right]\sigma_{M,t}^2 + \left[\frac{-J_{WF}}{J_W}\right]\sigma_{MF,t} \tag{2.1}$$

where $J(W(t), F(t), t)$ is the indirect utility function in wealth, $W(t)$, and $F(t)$ describing the evolution of the investment opportunity set over time; subscripts denote partial derivatives, and $\left[\frac{-J_{WW}W}{J_W}\right]$ is the coefficient of relative risk aversion, denoted as λ_M, which is typically assumed to be positive. The $\left[\frac{-J_{WF}W}{J_W}\right]$ in the second component describes the hedging coefficient. The sign of the hedging coefficient is indeterminate because it depends on the relationship between the marginal utility of wealth and the state of the world, and the conditional covariance. If the investment opportunity set is time-invariant, Merton (1980) shows that the hedging component is negligible and the conditional excess market return is proportional to its conditional variance.

$$E_t[r_{M,t+1} - r_{f,t}] = \left[\frac{-J_{WW}W}{J_W}\right]\sigma_{M,t}^2 \tag{2.2}$$

Since Merton (1980), this conditional CAPM specification has been subject to dozens of empirical investigations. In this paper, I first estimate the following univariate GARCH-M model (Model 1) used in Lundblad (2007).

$$r_{M,t+1} - r_{f,t} = \lambda_0 + \lambda_M\sigma_{M,t}^2 + \varepsilon_{t+1} \tag{2.3}$$

where ε_{t+1} is mean zero with conditional variance $(\sigma_{M,t}^2)$, $\sigma_{M,t+1}^2 = \delta_0 + \delta_1\varepsilon_t^2 + \delta_2\sigma_{M,t}^2$. $r_{M,t+1} - r_{f,t}$ is the stock market return in excess of the conditionally risk free rate.

Lundblad (2007) finds that in this specification, λ_M is positive and statistically significant with a long span of U.S. data. λ_0 is added to account for transaction costs or taxes. In preliminary empirical investigations, I experimented with various asymmetric GARCH specifications but the asymmetric terms are statistically insignificant for both stock and bond returns data. Therefore, I present empirical results only with the usual symmetric GARCH specifications.

Scruggs (1998) argues that the partial relationship between market risk premia and conditional volatility can be masked in the univariate context by failing to account for the additional hedging demands associated with a time varying investment opportunity set. Based on this observation, I estimate the following time-invariant two-factor ICAPM:

$$E_t\left[r_{M,t+1} - r_{f,t}\right] = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t}$$

Following Scruggs (1998) and Scruggs and Glabadanidis (2003) I use the long term bond return in excess of the risk free as a proxy for hedging portfolios. Specifically, I employ the following bivariate GARCH-M (Model 2). I constrain the prices of risk and hedging coefficient to be identical across markets consistent with the ICAPM theory.

$$
\begin{aligned}
r_{M,t+1} - r_{f,t} &= \lambda_{0,M} + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1} \\
r_{F,t+1} - r_{f,t} &= \lambda_{0,F} + \lambda_M \sigma_{MF,t}^2 + \lambda_F \sigma_{F,t}^2 + \varepsilon_{F,t+1} \\
\text{cov}\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1} | \psi_t\right] &= \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}
\end{aligned}
\tag{2.4}
$$

where $r_{F,t+1} - r_{f,t}$ is the long term bond return in excess of the conditionally risk free rate. ψ_t is the information set up to time t.

To describe the time-series evolution of the stock and bond market return conditional covariance matrix, I employ the following diagonal BEKK (1,1) specification.

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t}^2 & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon_{F,t}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

This specification guarantees the positive definiteness of the conditional covariance matrix, and yet allows time variation in conditional variances, covariances, and correlations across these markets. Consistent with the univariate GARCH-M analysis of the market portfolio returns and bond returns, I do not include asymmetric terms.

Finally, I generalize the ICAPM with the time-varying risk-return tradeoff and the changing hedging coefficient. Lundblad (2007) provides some preliminary evidence to show that the fundamental risk-return relationship has changed over time. While he presents evidence only within univariate context, I employ the following bivariate model allowing both time-varying risk-return relation and hedging coefficient (Model 3).

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_{M,t}\sigma_{M,t}^2 + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1}$$
$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_{M,t}\sigma_{MF,t}^2 + \lambda_{F,t}\sigma_{F,t}^2 + \varepsilon_{F,t+1}$$
$$\text{cov}\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}|\psi_t\right] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix} \qquad (2.5)$$

where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_m^2)$, $\lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma_f^2)$, Σ_t follows the same BEKK(1,1) as in the two-factor ICAPM.

A driftless random walk coefficient specification is often used to capture a persistent yet slow movement in time-varying coefficient models with stochastic volatility (e.g. Cogley and Sargent (2005)). I follow this tradition to facilitate the estimation with

parsimonious empirical models.

3 Estimation Methods for a Nonlinear ICAPM

I present the estimation framework for the proposed nonlinar ICAPM (Model 3) as the following state space model with GARCH terms.

Measurement equation:

$$
\begin{bmatrix} r_{M,t} - r_{f,t-1} \\ r_{F,t} - r_{f,t-1} \end{bmatrix} = \begin{bmatrix} \sigma^2_{M,t-1} & \sigma_{MF,t-1} \\ \sigma_{MF,t-1} & \sigma^2_{F,t-1} \end{bmatrix} \begin{bmatrix} \lambda_{M,t-1} \\ \lambda_{F,t-1} \end{bmatrix} + \begin{bmatrix} \lambda_{0,M} \\ \lambda_{0,F} \end{bmatrix} + \begin{bmatrix} \varepsilon_{M,t} \\ \varepsilon_{F,t} \end{bmatrix}
$$
$$
\begin{bmatrix} \varepsilon_{M,t} \\ \varepsilon_{F,t} \end{bmatrix} |\psi_{t-1} \sim N \left(0_2 \quad \begin{pmatrix} \sigma^2_{M,t-1} & \sigma_{MF,t-1} \\ \sigma_{MF,t-1} & \sigma^2_{F,t-1} \end{pmatrix} \right)
$$

In matrix terms, $y_t = \Sigma_{t-1}\beta_{t-1} + A + \varepsilon_t, \varepsilon_t \sim N(0, \Sigma_{t-1})$

where ψ_{t-1} is the information set up to time t-1. $\Sigma_t = \begin{pmatrix} \sigma^2_{M,t} & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma^2_{F,t} \end{pmatrix}$ with diagonal BEKK (1,1), y_t is a 2 x 1 vectors of returns observed at time t; β_t is a 2 x 1 vector of unobserved state variables; Σ_{t-1} is a 2 x 2 matrix that links the observed vector y_t and the unobserved β_t ; A is a 2 x 1 constant vector.

Transition equation:
$$
\begin{bmatrix} \lambda_{M,t} \\ \lambda_{F,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{M,t-1} \\ \lambda_{F,t-1} \end{bmatrix} + \begin{bmatrix} \omega_{M,t} \\ \omega_{F,t} \end{bmatrix}, \begin{bmatrix} \omega_{M,t} \\ \omega_{F,t} \end{bmatrix} \sim N \left(0_2 \quad \begin{pmatrix} \sigma^2_m & 0 \\ 0 & \sigma^2_f \end{pmatrix} \right)
$$

In matrix terms, $\beta_t = F\beta_{t-1} + \omega_t, \omega_t \sim N(0, Q)$

where β_t is a 2 x 1 vector of unobserved state variables; F is 2 x 2 ; ω_t is 2 x 1.

Following Harvey, Ruiz, and Sentana (1992), I augment the heteroskedastic shocks into the original state vector in the transition equation to get the conditional expectations of the squares of the unobserved shocks. The transformed state space model has modified equations as follows.

Measurement equation:

$$
\begin{bmatrix} r_{M,t} - r_{f,t-1} \\ r_{F,t} - r_{f,t-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \sigma^2_{M,t-1} & \sigma_{MF,t-1} & 1 & 0 \\ 0 & 0 & \sigma_{MF,t-1} & \sigma^2_{F,t-1} & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{M,t} \\ \lambda_{F,t} \\ \lambda_{M,t-1} \\ \lambda_{F,t-1} \\ \varepsilon_{M,t} \\ \varepsilon_{F,t} \end{bmatrix} + \begin{bmatrix} \lambda_{0,M} \\ \lambda_{0,F} \end{bmatrix}
$$

In matrix terms, $y_t = H_t^* \beta_t^* + A^*$.

Transition equation:

$$
\begin{bmatrix} \lambda_{M,t} \\ \lambda_{F,t} \\ \lambda_{M,t-1} \\ \lambda_{F,t-1} \\ \varepsilon_{M,t} \\ \varepsilon_{F,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{M,t-1} \\ \lambda_{F,t-1} \\ \lambda_{M,t-2} \\ \lambda_{F,t-2} \\ \varepsilon_{M,t-1} \\ \varepsilon_{F,t-1} \end{bmatrix} + \begin{bmatrix} \omega_{M,t} \\ \omega_{F,t} \\ \lambda_{M,t-1} \\ \lambda_{F,t-1} \\ \varepsilon_{M,t} \\ \varepsilon_{F,t} \end{bmatrix}
$$

$$
\begin{bmatrix} \omega_{M,t} \\ \omega_{F,t} \\ \lambda_{M,t-1} \\ \lambda_{F,t-1} \\ \varepsilon_{M,t} \\ \varepsilon_{F,t} \end{bmatrix} \sim N \left(0_6 \begin{pmatrix} \sigma_m^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_f^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{M,t-1}^2 & \sigma_{MF,t-1} \\ 0 & 0 & 0 & 0 & \sigma_{MF,t-1} & \sigma_{F,t-1}^2 \end{pmatrix} \right)
$$

In matrix terms, the transformed transition equation is stated as $\beta_t^* = F^* \beta_{t-1} + \omega_t^*, \omega_t^* \sim N(0, Q_t^*)$

Given the model's parameters, the linear Kalman filter for the state-space model consists of the following six equations:[2]

Prediction:

$$
\beta_{t|t-1}^* = F^* \beta_{t-1|t-1}^*,
$$

$$
p_{t|t-1}^* = F^* p_{t-1|t-1}^* F^{*\prime} + Q_t^*,
$$

$$
\eta_{t|t-1}^* = y_t - H_t^* \beta_{t|t-1}^* - A^*,
$$

$$
f_{t|t-1}^* = H_t^* p_{t|t-1}^* H_t^{*\prime},
$$

where $\beta_{t|t-1}^* = E[\beta_t^* | \psi_{t-1}]$, $\beta_{t-1|t-1}^* = E[\beta_{t-1}^* | \psi_{t-1}]$, $p_{t|t-1}^* = E[(\beta_t^* - E[\beta_t^* | \psi_{t-1}])^2]$, $p_{t-1|t-1}^* = E[(\beta_{t-1}^* - E[\beta_{t-1}^* | \psi_{t-1}])^2]$, $\eta_{t|t-1}^* = y_t - E[y_t | \psi_{t-1}]$, $f_{t|t-1}^* = E[(\eta_{t|t-1}^*)^2]$.

Updating:

$$
\beta_{t|t}^* = \beta_{t|t-1}^* + p_{t|t-1}^* H_t^{*\prime} f_{t|t-1}^{*-1} \eta_{t|t-1}^*,
$$

$$
p_{t|t}^* = p_{t|t-1}^* - p_{t-1|t-1}^* H_t^{*\prime} f_{t|t-1}^{*-1} H_t^* p_{t|t-1}^*,
$$

where $\beta_{t|t}^* = E[\beta_t^* | \psi_t]$, $p_{t|t}^* = E[(\beta_t^* - E[\beta_t^* | \psi_t])^2]$.

To process the above Kalman filter, following Harvey, Ruiz, and Sentana (1992), I approximate $\varepsilon_{M,t-1}^2$ $(\varepsilon_{F,t-1}^2)$ with $E[\varepsilon_{M,t-1}^2 | \psi_{t-1}]$ $(E[\varepsilon_{F,t-1}^2 | \psi_{t-1}])$ respectively in the Σ_{t-1}

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[2]I closely follow the notations in the chapter 6 of Kim and Nelson (1999).

matrix.

Given the fact that $\varepsilon_{M,t-1} = E[\varepsilon_{M,t-1}|\psi_{t-1}] + \varepsilon_{M,t-1} - E[\varepsilon_{M,t-1}|\psi_{t-1}]$, $\varepsilon_{F,t-1} = E[\varepsilon_{F,t-1}|\psi_{t-1}] + \varepsilon_{F,t-1} - E[\varepsilon_{F,t-1}|\psi_{t-1}]$, we compute $E[\varepsilon_{M,t-1}^2|\psi_{t-1}] = E[\varepsilon_{M,t-1}|\psi_{t-1}]^2 + E[(\varepsilon_{M,t-1} - E[\varepsilon_{M,t-1}|\psi_{t-1}])^2]$ and $E[\varepsilon_{F,t-1}^2|\psi_{t-1}] = E[\varepsilon_{F,t-1}|\psi_{t-1}]^2 + E[(\varepsilon_{F,t-1} - E[\varepsilon_{F,t-1}|\psi_{t-1}])^2]$ where $E[\varepsilon_{M,t-1}|\psi_{t-1}]$ and $E[\varepsilon_{F,t-1}|\psi_{t-1}]$ are obtained from the last two elements of $\beta_{t-1|t-1}{}^*$. $E[(\varepsilon_{M,t-1} - E[\varepsilon_{M,t-1}|\psi_{t-1}])^2]$ and $E[(\varepsilon_{F,t-1} - E[\varepsilon_{F,t-1}|\psi_{t-1}])^2]$ are obtained from the last two diagonal elements of $p_{t-1|t-1}{}^*$.

Finally, I approximate $\varepsilon_{M,t-1}\varepsilon_{F,t-1}$ as $(E[\varepsilon_{M,t-1}^2|\psi_{t-1}]E[\varepsilon_{F,t-1}^2|\psi_{t-1}])^{0.5}$. Because this last approximation of $\varepsilon_{M,t-1}\varepsilon_{F,t-1}$ implicitly assumes that two shocks should have the same signs, I should denote this model as an approximate BEKK model. However, the approximation errors are minimal using U.K. data in this paper. The GARCH variance and covariance estimates presented in the Section 5 for this model are almost same as those from a bivariate GARCH-M with BEKK specification.

As by-products of the above Kalman filter, I obtain the prediction error $\eta_{t|t-1}{}^*$ and its variance $f_{t|t-1}{}^*$. This prediction error decomposition induces the approximate log likelihood as follows.

$$\ln L = -\frac{1}{2}\sum_{t=1}^{T}\ln((2\pi)^n|f_{t|t-1}{}^*|) - \frac{1}{2}\sum_{t=1}^{T}\eta_{t|t-1}{}^{*'}f_{t|t-1}{}^{*-1}\eta_{t|t-1}{}^*$$

which can be maximized with respect to the unknown parameters of the model for an approximate Quasi-MLE.

4 Empirical Analysis

4.1 Data Description

As Lundblad (2007) carefully demonstrates, we need a long span of data to investigate the risk-return trade-off to enhance the power of the time-series analysis. To maximize the power of the time-series analysis, I employ the longest monthly UK equity and bond market data from 1836:01 to 2010:12. Earlier U.K. data extending back to 1800 are also available, but I exclude this sample because the stock market data only represent a simple equal weighted average of three shares: the Bank of England, the East India Company, and the South Sea Company. For a more detailed explanation on the data sources, see the documentation from www.globalfindata.com.

The UK historical stock data (ticker symbol: TFTASD) are taken from the Global Financial Data provider, and represent the FTSE All Shares historical index. I also collect total return data for the UK short-term bill (ticker symbol: TRGBRBIM) and long-term consol bond (ticker symbol: TRGBRGCM) from the same provider. Short-term bill data will serve as the conditionally risk-free rate in my analysis. As suggested by Merton (1973) and Scruggs (1998), I collect long-term U.K. bond returns to capture variation in the investment opportunity set over time.

Table 1 reports summary statistics on the total returns for the U.K. equity market, $r_{M,t}$, the bond market, $r_{F,t}$, and the short bill return (the conditionally risk-free rate), $r_{f,t}$, for the full sample. All variables are expressed as continuously compounded returns. The return data for each series are also displayed in Figure 1. In the whole sample, the mean return on the U.K. stock market portfolio is about 0.57% per month. As expected, the stock market return is highly volatile (3.6% per month). Long term bond and short term bill returns have similar lower mean return around 0.36% per month and also lower volatility as expected.

4.2 Expected Return - Volatility Tradeoff

4.2.1 The Conditional CAPM

Many previous studies on the risk-return tradeoff employ a univariate GARCH-M framework. I reproduce this model (Model 1) for an easier explanation.

$$E_t\left[r_{M,t+1} - r_{f,t}\right] = \lambda_0 + \lambda_M \sigma_{M,t}^2, \sigma_{M,t+1}^2 = \delta_0 + \delta_1 {\varepsilon_t}^2 + \delta_2 \sigma_{M,t}^2$$

In summary, these studies typically find a statistically insignificant or a negative relationship between the market risk premium and its expected volatility. A notable exception is Lundblad (2007). Using simulations, he demonstrates that even 100 years of data constitute a small sample that may easily lead to this puzzling insignificant or negative risk-return relation even though the true risk-return tradeoff is positive. Using the nearly two century history of US equity market returns, Lundblad estimates a positive and statistically significant risk-return tradeoff across every specification considered. While he also presents similar evidence with UK data, I find that the evidence might not be robust.

Table 2 presents evidence on the risk-return tradeoff in the univariate context with a long span of U.K. data from 1836:01 to 2010:12. I use continuously compounded returns as in Scruggs (1998) and report the estimates with the usual symmetric GARCH-M because I find that asymmetric terms in the GARCH specifications are not statistically significant.[3] The point estimate, presented in panel A, is 1.66 with a t statistics of 1.81, which is quite different from Lundblad's estimate using simple returns (2.469 with a standard error of 0.906 in his table). To reconcile the difference, I also estimate the same model with simple returns, and the estimates are provided in panel B. The mean variance

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[3]Results are available upon request. Table 4 in Lundblad (2007) also presents evidence that asymmetric GARCH models are unnecessary for U.K. data.

tradeoff becomes positive (2.2522) and statistically significant. Because most papers in this literature such as Scruggs (1998) employ continuously compounded returns, these conflicting results from different ways of computing returns warrant further scrutiny.

Figure 2 displays the estimates of conditional market variance implied by the GARCH-M model. Elevated volatility is most pronounced during the Great Depression, 1973-74 stock market crash, the recent global financial crises of the late 1990. This volatility pattern seems to capture historical episodes well. For example, during the 1973-74 stock market crash, London Stock Exchange's FT 30 lost 73% of its value. The UK went into recession in 1974, with GDP falling by 1.1 %. At the time, the UK's property market was going through a major crisis, and a secondary banking crisis forced the Bank of England to bail out a number of lenders. After the definitive market low for the FT30 Index on January 6th 1975 when the index closed at 146, the market almost doubled over next 3 months.

4.2.2 The Intertemporal CAPM

Scruggs (1998) argues that the partial relationship between risk premia and conditional volatility can be masked in the univariate context by failing to account for the additional hedging demands associated with a time varying investment opportunity set (essentially generating an omitted variable bias). I will explore this issue in this section.

I estimate a bivariate GARCH-M model with BEKK (1,1) based on Scruggs and Glabadanidis (2003) with the conditional mean of the market portfolio excess return and the bond market excess return implied by the ICAPM. For the easier explanation, I reproduce the empirical model below (Model 2).

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1}$$

$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_M \sigma_{MF,t}^2 + \lambda_F \sigma_{F,t}^2 + \varepsilon_{F,t+1}$$

$$\text{cov}\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1} | \psi_t\right] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma_{M,t}^2 & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma_{F,t}^2 \end{pmatrix}$$

Consistent with the univariate GARCH-M analysis of the market portfolio returns and bond returns[4], I do not include asymmetric terms.

Table 3 presents estimation results for the two-factor ICAPM with diagonal BEKK specification. First, the conditional variances for the bond and stock markets are persistent (above 0.8) in the full historical record. Figure 3 presents time-series plots of the conditional stock and bond return heteroskedasticities and the conditional covariance. The equity variance displays similar patterns presented in the univariate model (see Figure 2). The bond variance is quite low compared with the stock variance but it increases dramatically after 1980's and especially during the recent crisis. Table 3 also present evidence on the intertemporal relationship between risk and expected return. In Panel A, for the whole sample, the partial relationships between the expected market excess return and market conditional variance is positive and statistically significant with t statistics of 2.09. However, the partial relationship between expected market excess returns and covariance with variation in the investment opportunity set is not statistically significant. Strictly speaking, a two factor ICAPM does not seem to be supported by the data.

4.2.3 A Time-varying risk-return Tradeoff?

There are several concerns associated with the potentially strong assumption of a time-invariant risk-return tradeoff. First, in equilibrium, the mean variance tradeoff can be

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[4]The results of univariate GARCH-M for bond excess returns are available upon request

interpreted as risk aversion and may exhibit cyclical variation through time as implied by habit models such as Campbell and Cochrane (1999). Moreover, risk aversion may also vary because of the evolution of financial markets and improved risk sharing. Motivated by these observations, I conduct an empirical analysis of a time-varying risk-return tradeoff, and investigate whether the proposed new nonlinear ICAPM shed light on the puzzling risk-return relation.

Lundblad (2007) also conducts an exploratory analysis to allow the risk-return trade-off coefficient to vary with several observable financial and macroeconomic indicators of the development of the U.S. financial market and economy. However, Lundblad employs only univariate models in this context. Without fully applying time-varying coefficients in the risk-return tradeoff, it is unclear if the time-varying relation exist or it just indicates misspecifications. Due to the limit of historical data, I choose econometric models with latent factors to estimate the unstable relation rather than to use the models with exogenous variables. Before conducting a formal econometric analysis in the next section, I present preliminary evidence of instability by estimating the two-factor time-invariant ICAPM recursively at least with 10 years of data starting from 1846:01. Figure 4 presents these rolling sample estimates of the risk-return tradeoff and the hedging co-efficient from the ICAPM. While it is difficult to argue for the time-varying risk-return relation with any formal statistics at this stage, it looks as if both the risk-return tradeoff and the hedging coefficients vary a lot.

5 A Time-varying risk-return Tradeoff with a Non-linear ICAPM

Motivated by theoretical arguments and preliminary rolling estimates of the ICAPM, I estimate a nonlinear ICAPM with the time-varying risk-return tradeoff and hedging

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coefficients (Model 3). I reproduce the empirical model for an easier explanation.

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_{M,t}\sigma^2_{M,t} + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1}$$

$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_{M,t}\sigma^2_{MF,t} + \lambda_{F,t}\sigma^2_{F,t} + \varepsilon_{F,t+1}$$

$$\text{cov}\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}|\psi_t\right] = \Sigma_t, \Sigma_t = \begin{pmatrix} \sigma^2_{M,t} & \sigma_{MF,t} \\ \sigma_{MF,t} & \sigma^2_{F,t} \end{pmatrix}$$

where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2_m)$, $\lambda_{F,t} = \lambda_{F,t-1} + \eta_t$, $\eta_t \sim N(0, \sigma^2_f)$.

Table 4 presents parameter estimates of this nonlinear ICAPM. First, I find that variance estimates in the time-varying risk-return (σ_v) and hedging coefficients (σ_F) are statistically significant at 5% level. This evidence is reassuring because it supports the time-varying risk-return trade off and the changing hedging coefficient assumed in the nonlinear ICAPM. Figure 5 presents time-series plots of the conditional stock and bond return variances and the conditional stock-bond return covariance estimated from this model. The stock return variance follows the same patterns presented in the conditional CAPM and ICAPM (see Figure 2 and 3). The bond variance displays clustering consistent with the estimates from ICAPM (see Figure 3) as well, increasing dramatically during the early 1980's. Over the full historical record, the conditional covariance between the stock and bond market is a small positive number.

Figure 6 shows the time-varying risk-return relation and 95% confidence bands estimated from the nonlinear ICAPM. The estimated partial expected market return volatility tradeoff is largely positive and statistically significant for the full historical record. Further, this figure shows that the seemingly negative relation could be entirely spurious because the estimated relation is not statistically different from zeros with the 95% confidence bounds. The estimated hedging coefficient is also time-varying and negative over the time (Figure 7). This evidence indicates that incorporating time-varying risk-return relation and the changing coefficient in the hedging demands associated with variation in

the investment opportunity set is crucial to understand the puzzling risk-return tradeoff. Contrary to the weak risk-return tradeoff provided with the time-invariant conditional CAPM and ICAPM, I find that the expected market return conditional volatility tradeoff is positive and statistically significant for the full historical record.

6   Conclusion

While the risk-return tradeoff is fundamental to finance, the empirical evidence on the relationship between the risk premium on aggregate stock market and the variance of its return is ambiguous at best. Lundblad (2007) argues the main culprit of this puzzling relationships is the small sample problem. He finds a statistically significant positive risk-return tradeoff using information from two century history of stock market returns in all of the econometric specifications.

In this paper, I present new evidence that the risk-return trade-off is rather weak even with the two century history of UK continuously compounded return data, when I employ a time-invariant conditional CAPM or two-factor ICAPM. In the conditional CAPM, the risk-return tradeoff parameter is positive yet statistically insignificant at 5% level. When the time-varying investment opportunity set is explicitly accounted as the hedging component, the risk-return tradeoff becomes positive and statistically significant at 5% level. But one of the crucial implications of the ICAPM is rejected; the hedging coefficient is insignificant even at 10% level.

Motivated by theoretical arguments and preliminary rolling estimates of the ICAPM, I develop and estimate a nonlinear ICAPM with the time-varying risk-return tradeoff and hedging coefficient. Consistent with the implication of the model, I find that variance estimates in the time-varying risk-return and hedging coefficients are statistically significant at 1% level.

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The estimated risk-return relation in this nonlinear ICAPM becomes largely positive over the time. While the negative risk-return relation can be certainly observed, it is statistically insignificant with the 95% confidence bounds. This complex nonlinearity seems to be the main culprit of the weak risk-return relation observed in the literature.
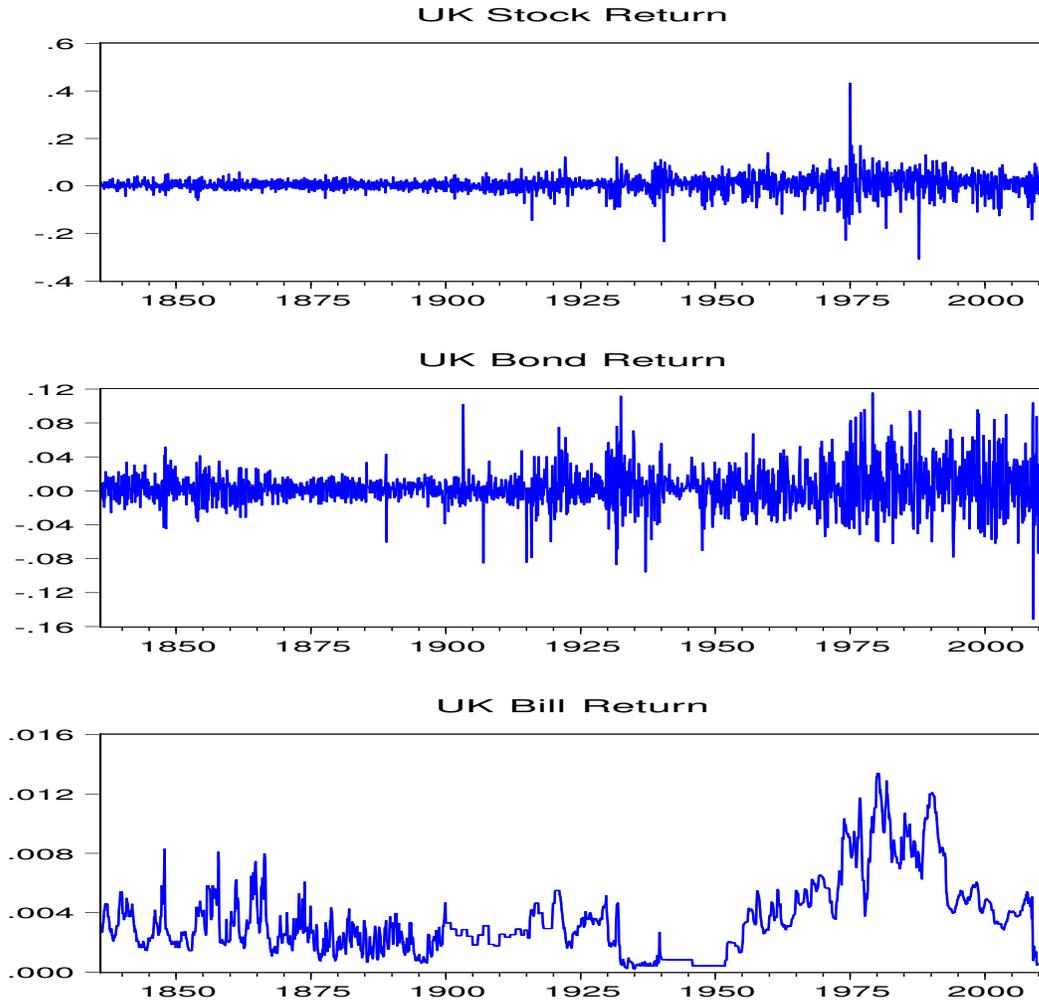
This figure plots the monthly time series of returns on U.K. stock market portfolios ($r_{M,t}$), long term bond returns ($r_{F,t}$), and the return on the short term bill ($r_{f,t}$) are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.
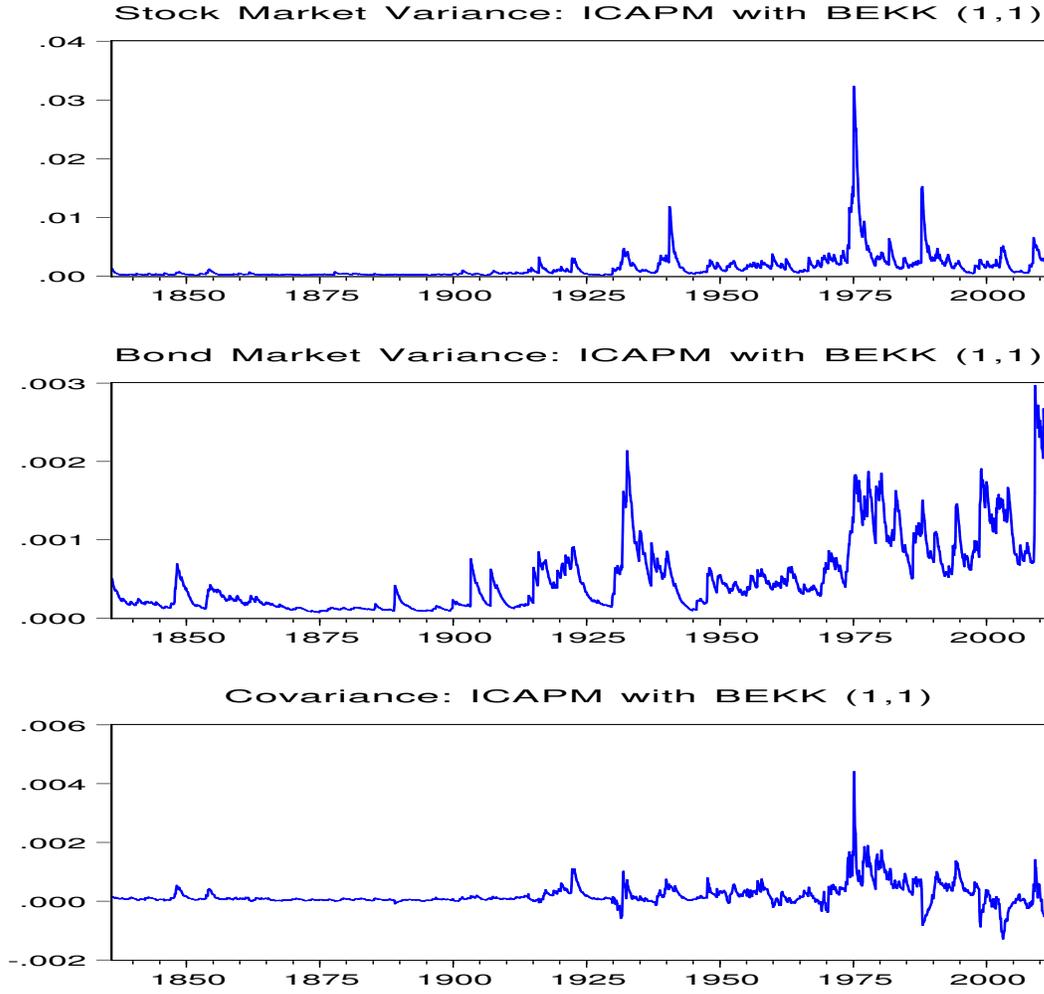
Figure 1: Time Series Plots of Asset Returns (1836:01 - 2010:12)

Stock Market Variance: CAPM with GARCH-M (1,1)
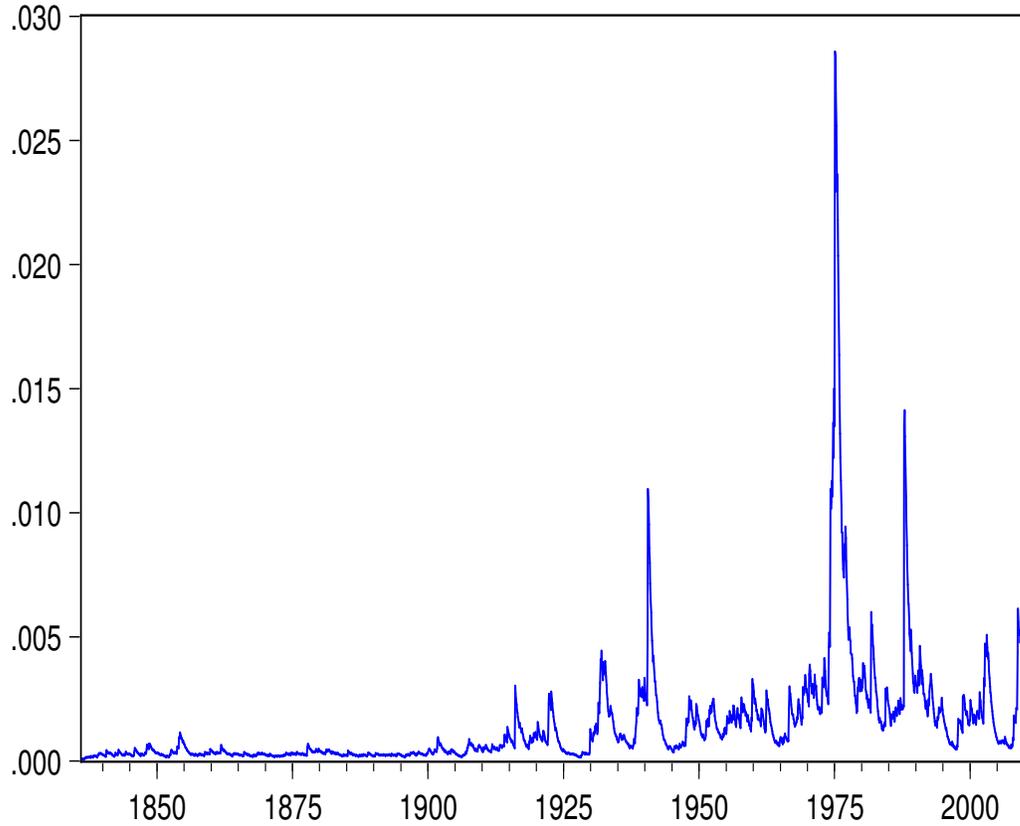
This figure plots the monthly time series of conditional variance of the U.K. stock market excess returns $(r_{M,t}\text{-}r_{f,t})$ estimated by the GARCH-M from 1836:01 to 2010:12. Returns on U.K. stock market portfolios $(r_{M,t})$ and the return on the short term bill $(r_{f,t})$ are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns. The mean equation is: $r_{M,t+1} - r_{f,t} = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \varepsilon_{t+1}$, where ε_{t+1} is mean zero with the conditional variance $(\sigma_{M,t}^2)$, $\sigma_{M,t+1}^2 = \delta_0 + \delta_1 \varepsilon_t^2 + \delta_2 \sigma_{M,t}^2$

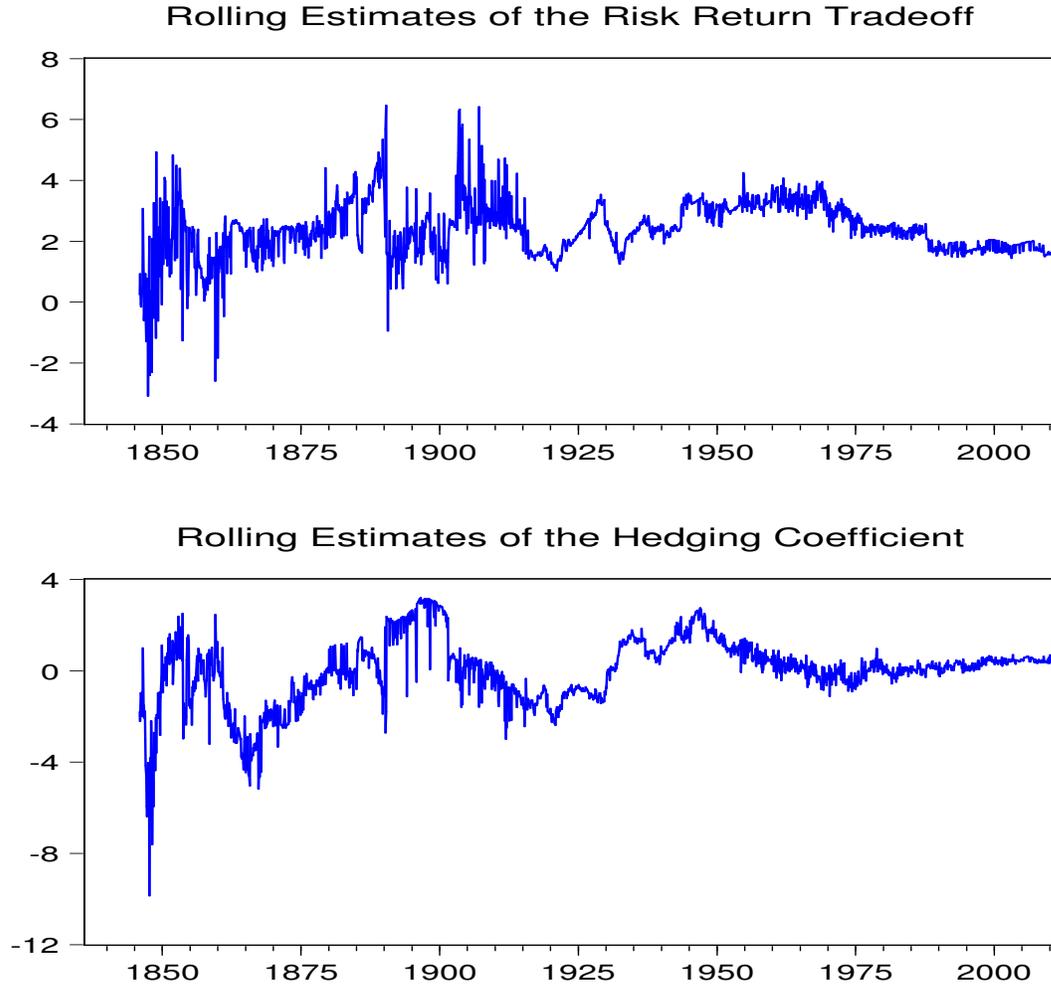Figure 2: Conditional Equity Variance from a Conditional CAPM

This figure plots the monthly time series of conditional variances of the U.K. stock market excess returns $(r_{M,t}\text{-}r_{f,t})$ and of the U.K. bond market excess returns $(r_{F,t}\text{-}r_{f,t})$ and their covariance estimated by the two-factor ICAPM with BEKK (1,1) from 1836:01 to 2010:12. Returns on U.K. stock market portfolios $(r_{M,t})$, long term bond returns $(r_{F,t})$, and the return on the short term bill $(r_{f,t})$ are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.

The two-factor ICAPM with BEKK (1,1):

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_M \sigma^2_{M,t} + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1}$$
$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_M \sigma^2_{MF,t} + \lambda_F \sigma^2_{F,t} + \varepsilon_{F,t+1}$$
$$\text{cov}_t \left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1} \right] = \Sigma_t$$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon^2_{M,t} & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon^2_{F,t} \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Figure 3: Conditional Covariance Matrix Estimates from a two-factor ICAPM

Rolling Estimates of the Risk Return Tradeoff

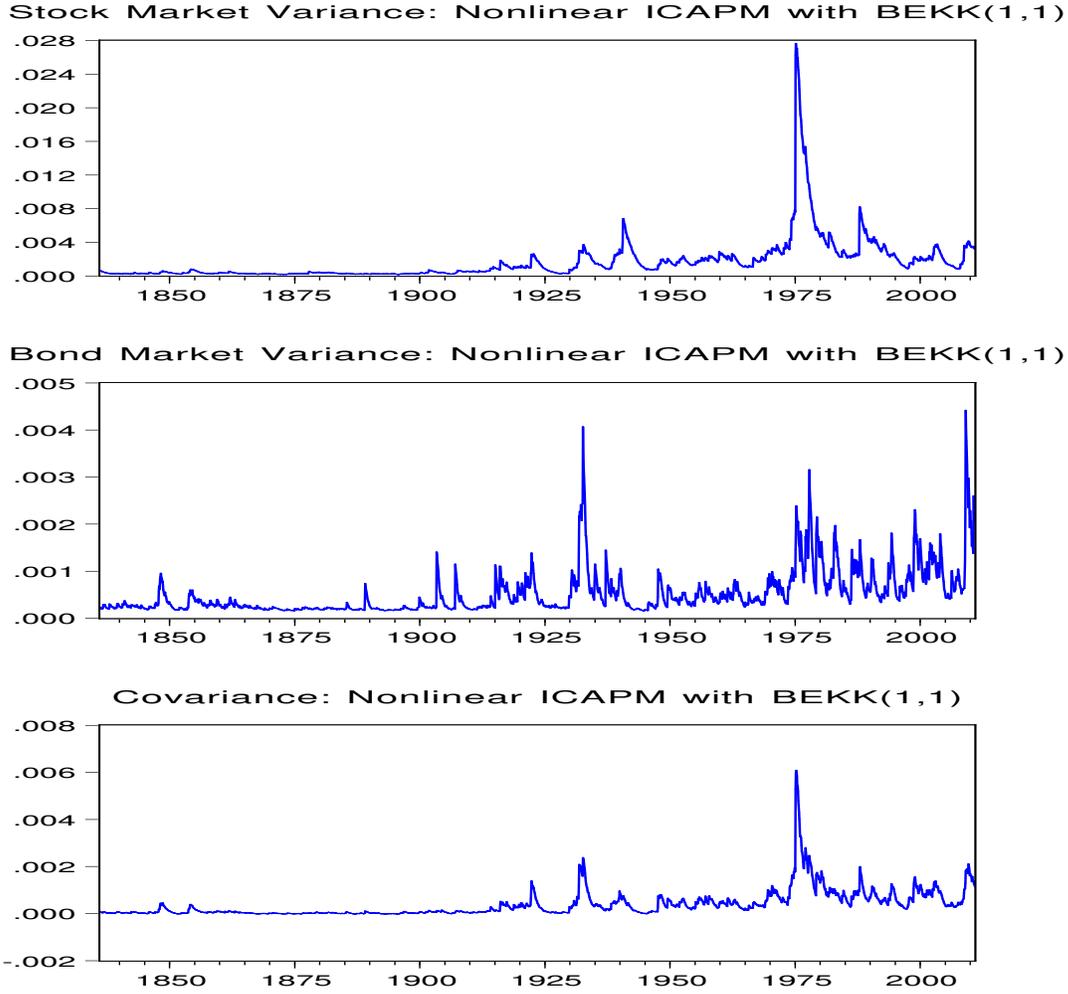
Rolling Estimates of the Hedging Coefficient

This figure plots the monthly time series of rolling estimates of the price of risk and hedging coefficients estimated by the two-factor ICAPM with BEKK (1,1) from 1836:01 to 2010:12. Returns on U.K. stock market portfolios $(r_{M,t})$, long term bond returns $(r_{F,t})$, and the return on the short term bill $(r_{f,t})$ are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.

The two-factor ICAPM with BEKK (1,1):

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1}$$
$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_M \sigma_{MF,t}^2 + \lambda_F \sigma_{F,t}^2 + \varepsilon_{F,t+1}$$
$$\text{cov}_t\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}\right] = \Sigma_t$$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t}^2 & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon_{F,t}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Figure 4: Rolling estimates of Risk Aversion and Hedging Coefficient

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Stock Market Variance: Nonlinear ICAPM with BEKK(1,1)

Bond Market Variance: Nonlinear ICAPM with BEKK(1,1)

Covariance: Nonlinear ICAPM with BEKK(1,1)

This figure plots the monthly time series of conditional variances of the U.K. stock market excess returns $(r_{M,t}\text{-}r_{f,t})$ and of the U.K. bond market excess returns $(r_{F,t}\text{-}r_{f,t})$ and their covariance estimated by the two-factor nonlinear ICAPM with BEKK (1,1) from 1836:01 to 2010:12. Returns on U.K. stock market portfolios $(r_{M,t})$, long term bond returns $(r_{F,t})$, and the return on the short term bill $(r_{f,t})$ are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.

The two-factor nonlinear ICAPM with BEKK (1,1):

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_{M,t}\sigma_{M,t}^2 + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1}$$
$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_{M,t}\sigma_{MF,t}^2 + \lambda_{F,t}\sigma_{F,t}^2 + \varepsilon_{F,t+1}$$
$$\text{cov}_t\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}\right] = \Sigma_t$$

where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_m^2)$ and $\lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma_f^2)$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t}^2 & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon_{F,t}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Figure 5: Conditional Covariance Matrix Estimates from a two-factor nonlinear ICAPM
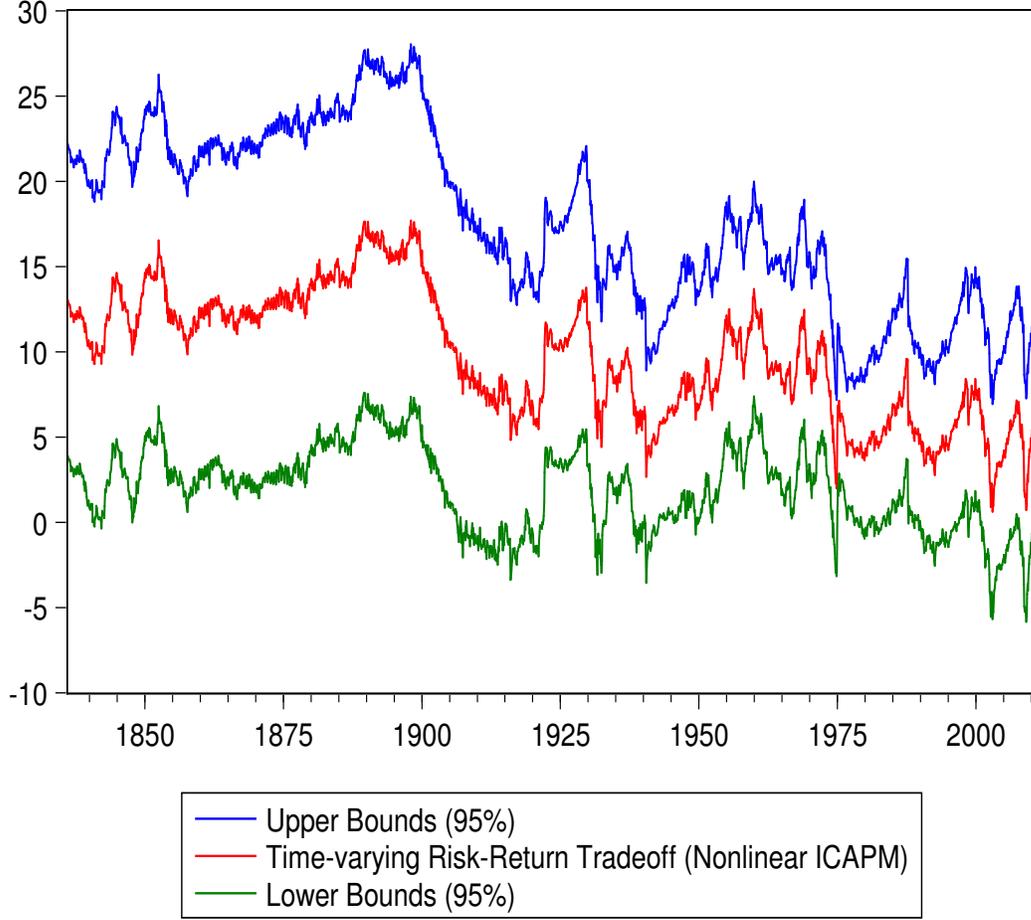
This figure plots the monthly time series of estimates of the time-varying price of risk and its confidence intervals estimated by the two-factor nonlinear ICAPM with BEKK (1,1) from 1836:01 to 2010:12. Returns on U.K. stock market portfolios ($r_{M,t}$), long term bond returns ($r_{F,t}$), and the return on the short term bill ($r_{f,t}$) are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.
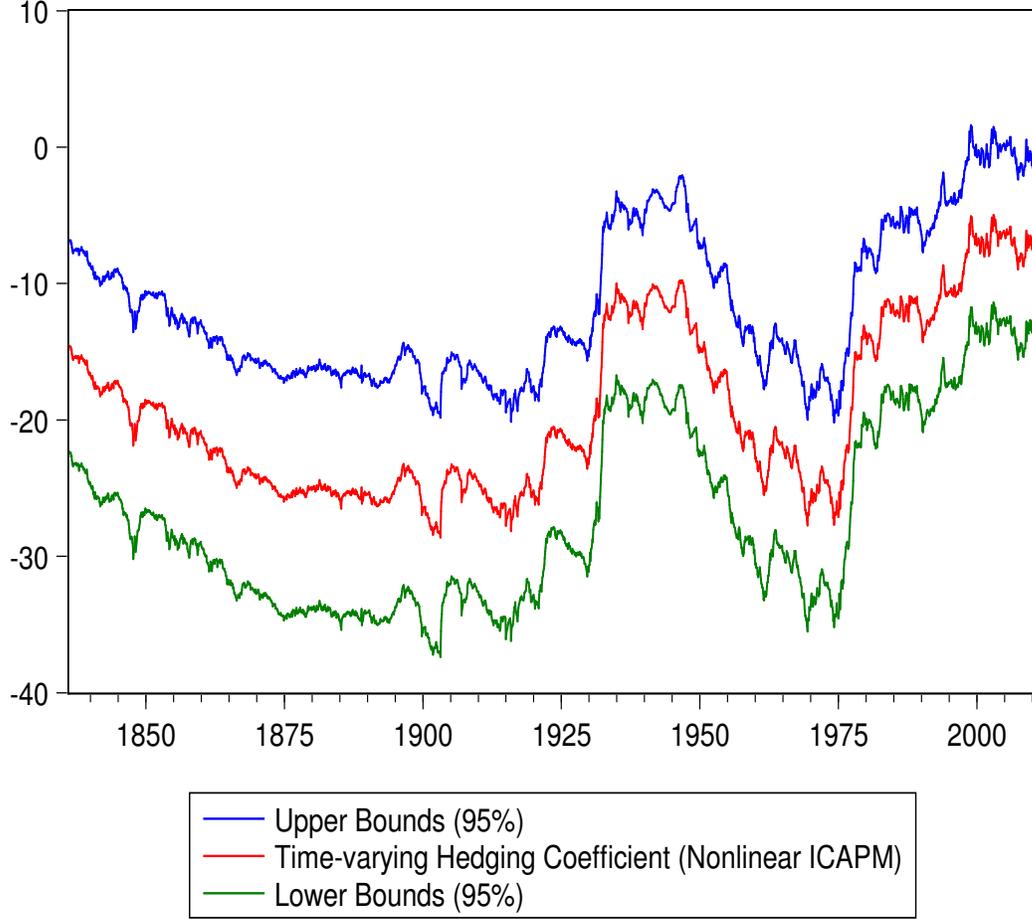
The two-factor nonlinear ICAPM with BEKK (1,1):

$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_{M,t}\sigma^2_{M,t} + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1}$

$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_{M,t}\sigma^2_{MF,t} + \lambda_{F,t}\sigma^2_{F,t} + \varepsilon_{F,t+1}$

$\text{cov}_t\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}\right] = \Sigma_t$

where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2_m)$ and $\lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma^2_f)$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon^2_{M,t} & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon^2_{F,t} \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$

$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Figure 6: Time-varying Risk-Return Tradeoff

This figure plots the monthly time series of estimates of the time-varying hedging coefficient and its confidence intervals estimated by the two-factor nonlinear ICAPM with BEKK (1,1) from 1836:01 to 2010:12. Returns on U.K. stock market portfolios ($r_{M,t}$), long term bond returns ($r_{F,t}$), and the return on the short term bill ($r_{f,t}$) are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.

The two-factor nonlinear ICAPM with BEKK (1,1):

$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_{M,t}\sigma_{M,t}^2 + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1}$

$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_{M,t}\sigma_{MF,t}^2 + \lambda_{F,t}\sigma_{F,t}^2 + \varepsilon_{F,t+1}$

$\text{cov}_t\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}\right] = \Sigma_t$

where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_m^2)$ and $\lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma_f^2)$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t}^2 & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon_{F,t}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$

$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Figure 7: Time-varying Hedging Coefficient

Table 1: Descriptive Statistics (1836:01 - 2010:12) for Asset Returns

This table reports summary statistics and autocorrelation and correlations for returns on U.K. stock market portfolios ($r_{M,t}$), long term bond returns ($r_{F,t}$), and the return on the short term bill ($r_{f,t}$) are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded monthly returns.

	$r_{M,t}$	$r_{F,t}$	$r_{f,t}$
Mean	0.0057	0.0037	0.0035
Std. Dev.	0.0365	0.0222	0.0025
Skewness	0.0094	0.3573	1.3743
Kurtosis	18.4920	7.4670	4.9298
Auto(1)	0.0870	0.1110	0.9850

	$r_{M,t}$	$r_{F,t}$	$r_{f,t}$
$r_{M,t}$	1.0000		
$r_{F,t}$	0.2334	1.0000	
$r_{f,t}$	0.0324	0.0688	1.0000

Table 2: Risk-Return Tradeoff: a Conditional CAPM

This table presents evidence on the conditional mean and volatility of the U.K. stock market portfolio implied by the GARCH-M from 1836:01 to 2010:12. Returns on U.K. stock market portfolios ($r_{M,t}$) and the return on the short term bill ($r_{f,t}$) are obtained from the Global Financial Data provider. Panel A (B) presents estimation results with continuously compounded returns (simple returns). The mean equation is: $r_{M,t+1} - r_{f,t} = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \varepsilon_{t+1}$, where ε_{t+1} is mean zero with the conditional variance $(\sigma_{M,t}^2)$, $\sigma_{M,t+1}^2 = \delta_0 + \delta_1 \varepsilon_t^2 + \delta_2 \sigma_{M,t}^2$

	λ_0	λ_M	δ_0	δ_1	δ_2
Panel A	0.0004	1.6668	0	0.1088	0.8951
t-stat	0.7005	1.8132	2.5851	6.1362	64.8426
Panel B	0.0005	2.2522	0	0.1109	0.8918
t-stat	0.7151	2.2554	2.7842	6.2697	63.8226

Table 3: Risk Return Tradeoff in the ICAPM

This table provides parameter estimates for a two-factor ICAPM with the U.K. stock market excess returns $(r_{M,t}\text{-}r_{f,t})$ and the U.K. bond market excess returns $(r_{F,t}\text{-}r_{f,t})$ from 1836:01 to 2010:12. Returns on U.K. stock market portfolios $(r_{M,t})$, long term bond returns $(r_{F,t})$, and the return on the short term bill $(r_{f,t})$ are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.

The two-factor ICAPM with BEKK (1,1):

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \varepsilon_{M,t+1}$$
$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_M \sigma_{MF,t}^2 + \lambda_F \sigma_{F,t}^2 + \varepsilon_{F,t+1}$$
$$\text{cov}_t\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}\right] = \Sigma_t$$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon_{M,t}^2 & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon_{F,t}^2 \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Panel A. Conditional Mean			
		S.E.	t-stat
$\lambda_{0,M}$	0.0004	0.0006	0.7662
λ_M	1.3247	0.6350	2.0862
$\lambda_{0,F}$	-0.0002	0.0004	-0.5696
λ_F	0.98780	1.3478	0.7330

Panel B. Conditional Variance			
		S.E.	t-stat
c11	0.0030	0.0003	9.7926
c12	0.0006	0.0002	3.1362
c22	0.0015	0.0001	12.9421
a11	0.3565	0.0117	30.4533
a22	0.2388	0.0069	34.5690
b11	0.9377	0.0037	254.1151
b22	0.9702	0.0014	690.5052

Table 4: Risk Return Tradeoff in the Nonlinear ICAPM

This table provides parameter estimates for a two-factor nonlinear ICAPM with the U.K. stock market excess returns ($r_{M,t}$-$r_{f,t}$) and the U.K. bond market excess returns ($r_{F,t}$-$r_{f,t}$) from 1836:01 to 2010:12. Returns on U.K. stock market portfolios ($r_{M,t}$), long term bond returns ($r_{F,t}$), and the return on the short term bill ($r_{f,t}$) are obtained from the Global Financial Data provider. All variables are expressed as continuously compounded returns.

The two-factor nonlinear ICAPM with BEKK (1,1):

$$r_{M,t+1} - r_{f,t} = \lambda_{0,M} + \lambda_{M,t}\sigma^2_{M,t} + \lambda_{F,t}\sigma_{MF,t} + \varepsilon_{M,t+1}$$
$$r_{F,t+1} - r_{f,t} = \lambda_{0,F} + \lambda_{M,t}\sigma^2_{MF,t} + \lambda_{F,t}\sigma^2_{F,t} + \varepsilon_{F,t+1}$$
$$\text{cov}_t\left[\varepsilon_{M,t+1}, \varepsilon_{F,t+1}\right] = \Sigma_t$$

where $\lambda_{M,t} = \lambda_{M,t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2_m)$ and $\lambda_{F,t} = \lambda_{F,t-1} + \eta_t, \eta_t \sim N(0, \sigma^2_f)$

$$\Sigma_t = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}' \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}' \begin{pmatrix} \varepsilon^2_{M,t} & \varepsilon_{M,t}\varepsilon_{F,t} \\ \varepsilon_{M,t}\varepsilon_{F,t} & \varepsilon^2_{F,t} \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}' \Sigma_{t-1} \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

Panel A. Conditional Mean		S.E.	t-stat
$\lambda_{0,M}$	-0.0020	0.0013	-1.8227
$\lambda_{0,F}$	0.0058	0.0006	10.1628

Panel B. Conditional Variance		S.E.	t-stat
c11	0.0011	0.0003	4.0086
c12	0.0049	0.0004	11.2319
c22	0.0009	0.0015	0.6095
a11	0.2558	0.0036	72.0414
a22	0.3483	0.0210	16.6001
b11	0.9727	0.0010	1015.3340
b22	0.9138	0.0127	72.0047

Panel C. TVP variance		S.E.	t-stat
σ_v	0.4210	0.1159	3.6317
σ_F	0.2861	0.0730	3.9174

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